PhD Proposal in the field of Nonlinear Quantum graphs

Advisors : Colette De Coster (CERAMATHS - Valenciennes - France) and Louis Jeanjean (LMB - Besançon - France).

Subject of the PhD : *Existence and qualitative properties of solutions for some nonlinear equations set on metric graphs.*

This PhD grant is in the frame and is financed by the ANR Project NQG (ANR-23-CE40-0005-01) of the French National Research Agency (ANR). In particular an annual budget for attending conferences and workshops is planned.

Presentation of the project : The nonlinear Schrödinger equation is a ubiquitous model in physics, with many applications in fields as diverse as Bose-Einstein condensation or non-linear optics. In many physical situations, the underlying space (i.e. the domain on which the solutions of the equation are defined) is essentially one-dimensional and can be modeled as a *metric graph*, also called a *quantum graph*, i.e. a locally one–dimensional domain obtained by gluing together one–dimensional, possibly unbounded intervals, the *edges*, through the identification of some of their endpoints, the *vertices*. The mathematical study of this type of model is very recent and is growing incredibly. We refer to [2, 6] for a survey and to [1, 3, 7] for some particular works. We would also like to draw your attention to the upcoming Summer School : https://nqg.sciencesconf.org.

As when the non-linear Schrödinger equation is placed on the entire space the study of stationary solutions is the subject of special attention. We can then focus either on solutions with a given *frequency* or on solutions with a given *mass*. This second type of solutions is particularly interesting from a physical point of view (see [6]) because this quantity is preserved during the time evolution.

In both cases the questions that are asked regarding the existence and behavior of these solutions (as their orbital or asymptotic stability), depend on

(i) The type of graph considered (compact, non-compact, periodic, of tree type, ...).
(ii) The choice of vertex conditions (Kirchhoff, Dirac, Fulop-Tsutsui, ...).
(iii) The type of nonlinearity (mass subcritical, mass supercritical, local or non-local, focusing or defocusing).

According to the case we are in, different analytical methods can be put at work, such as variational or bifurcation ones.

The PhD will focus in particular on the so-called superlinear mass case and also on the existence of sign-changing solutions, the first results of which have only very recently been obtained [4, 5]. We also plan to examine equations other than the Schrödinger equation on a metric graph.

Conditions : The position is for a fixed term of three years and corresponds to a French *Contrat doctoral*. It should start this Autumn and no later than December 1 2024. The net salary will be around 1800 euros net per month. It may, but need not, be increased by teaching duties (in French). The candidate is expected to spend 18 months in Besançon and 18 months in Valenciennes. In both cases, on-campus accommodation should be available.

The application will contain :

- a motivation letter,
- a detailed curriculum vitae,
- certified grades for the three last academic years,
- name and email addresses of two references.

Please send your application or any further questions by mail with the subject **NQG PhD** both to Colette De Coster (Colette.DeCoster@uphf.fr) and Louis Jeanjean (louis.jeanjean@univ-fcomte.fr). The deadline for an application is April 20, 2024 and we except to give a final answer to the candidates by May 4, 2024.

[1] R. Adami, E. Serra and P. Tilli. NLS ground states on graphs. *Calc. Var. Partial Differential Equations* 54 (1): 743-761, 2015.

[2] G. Berkolaiko and P. Kuchment. *Introduction to quantum graphs*, Vol. 186 of *Mathematical Surveys and Monographs*. American Mathematical Society, Providence, RI, 2013.

[3] G. Berkolaiko, J. Marzuola, D. Pelinovsky. Edge-localized states on quantum graphs in the limit of large mass. *Ann. Inst. H. Poincaré C Anal. Non Linéaire* 38(5): 1295-1335, 2021.

[4] X. J. Chang, L. Jeanjean and N. Soave. Normalized solutions of L^2-supercritical NLS equations on compact metric graphs. *Ann. Inst. H. Poincaré C Anal. Non Linéaire*, (2023). https://doi.org/10.4171/AIHPC/88 and arXiv:2204.01043.

[5] C. De Coster, S. Dovetta, D. Galant, E. Serra, C. Troestler. Constant sign and sign changing NLS ground states on noncompact metric graphs : arXiv:2306.12121, 2023.

[6] A. Kairzhan, D. Noja and D. E. Pelinovsky. Standing waves on quantum graph. J. Phys. A: Math. Theor. 55 243001, 2022.

[7] D. Pierotti and N. Soave. Ground states for the NLS equation with combined nonlinearities on noncompact metric graphs. *SIAM J. Math. Anal.*, 54 (1): 768-790, 2022.